# Coding for Interactive Communication

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# Interactive Communication

**One-way communication**: one party wants to send a msg to the other.

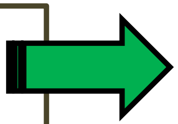
**Two-way (interactive) communication**:

Alice gets x∈ $\{0,1\}^k$, Bob gets y∈ $\{0,1\}^k$

Compute f(x,y) via many back-and-forth msg exchanges

**Coding for interactive communication**

$\Pi$: an n-round protocol for the noiseless setting

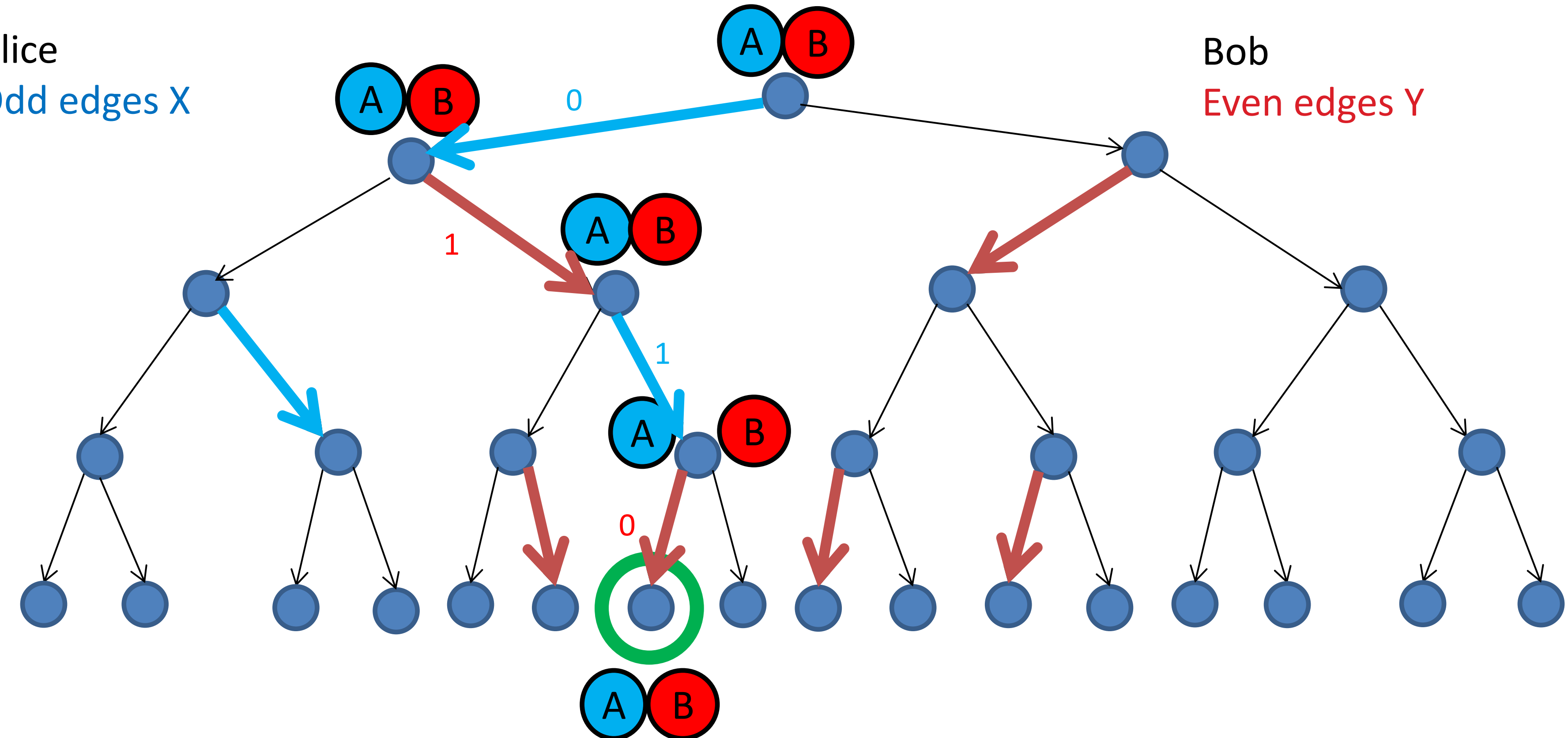


$\Pi'$: an N-round protocol that simulates $\Pi$ even if $\rho N$ transmissions are changed.

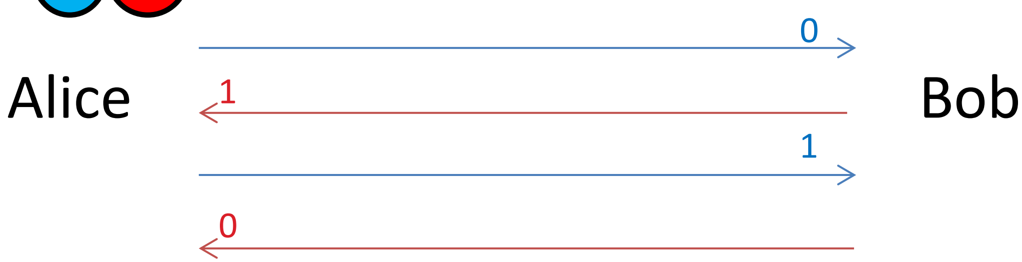
# Pointer Jumping

Alice
Odd edges X

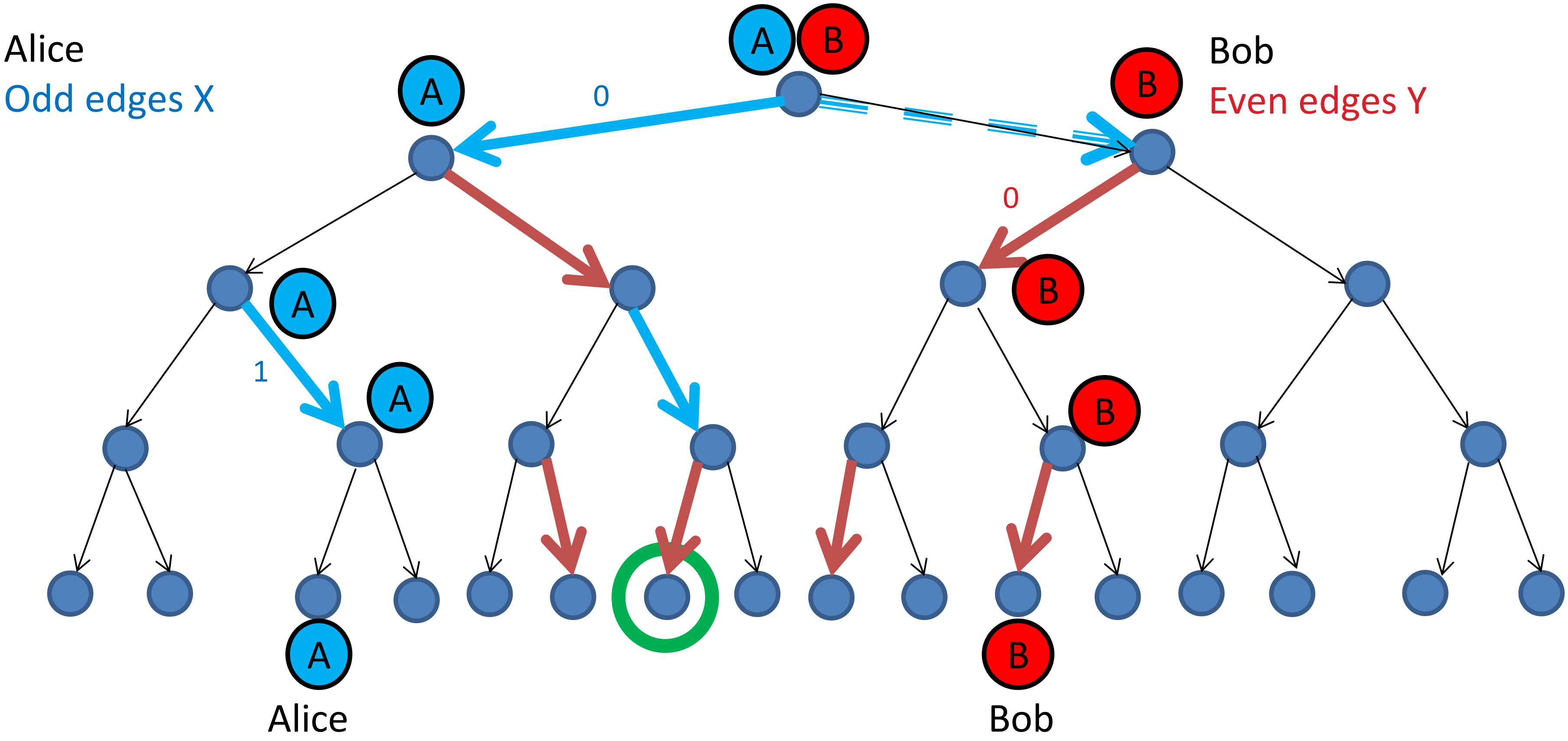
Bob
Even edges Y




Goal: Find the unique blue-red path



# Pointer Jumping with (adversarial) errors




undetected error ➔ Alice and Bob follow different parts of the tree.

Standard Error-Correcting Codes are not sufficient

# What's known? (adversarial error)

Focus: **Tolerable Error-Rate**

- Schulman FOCS'92, STOC'93: ${}^{1}/_{240} - \epsilon$

N=O(n) communication rounds, exp(n) computation

- Braverman & Rao STOC'11: ${}^{1}/_{4} - \epsilon$

N=O(n) communication rounds, exp(n) computation

Other measures: **communication complexity** & **computational complexity**

- Brakerski & Kalai FOCS'12: ${}^{1}/_{16} - \epsilon$,

N=O(n) communication rounds, $\tilde{O}(n^2)$ computation

- Brakerski & Naor SODA'13: unspecified Θ(1),

N=O(n) rounds, O(n log n) computation

**New:**

Tolerable error-rate ${}^{2}/_{7} - \epsilon$

Communication complexity N=O(n)

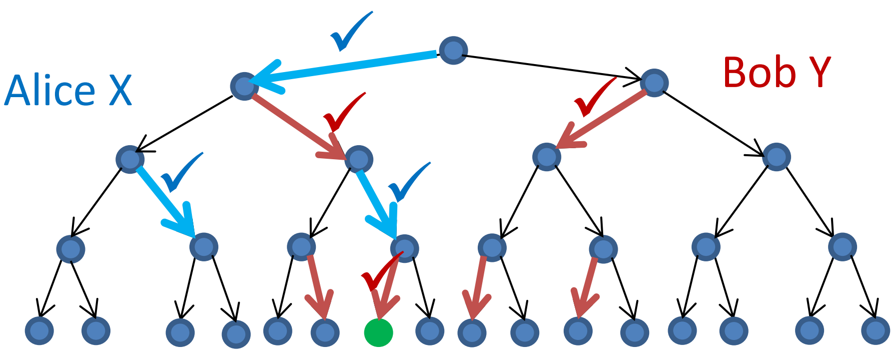
Computational Complexity $\tilde{O}(n)$

# Tolerating error-rate 1/4 - $\epsilon$

Take N=O(n/$\epsilon$) rounds

Alice $E_A \subseteq X$ , Bob $E_B \subseteq Y$

Grow $E_A$ and $E_B$ one edge at a time.




**Alice's Alg.**

<u>Sending round</u>: send one symbol indicating the whole $E_A$

using large O(n)-bit size alph. ➔ remedy: tree-codes

<u>Receiving round</u>: receive $E'_B$; ignore if it looks "invalid".

If $E_A \cup E'_B$ ends at a leaf v, add one vote to v.

Otherwise, if $E_A \cup E'_B$ can be extended along X via an edge e, let $E_A = E_A \cup \{e\}$.

# Tolerating error-rate 1/4 - $\epsilon$

Sending round: send a one-symbol encoding of (the whole) $E_A$

Receiving round: suppose received $E'_B$; ignore if it looks "invalid".

If $E_A \cup E'_B$ ends at a leaf v, add one vote to v.

Otherwise, if $E_A \cup E'_B$ can be extended along X with edge e, let $E_A = E_A \cup \{e\}$.

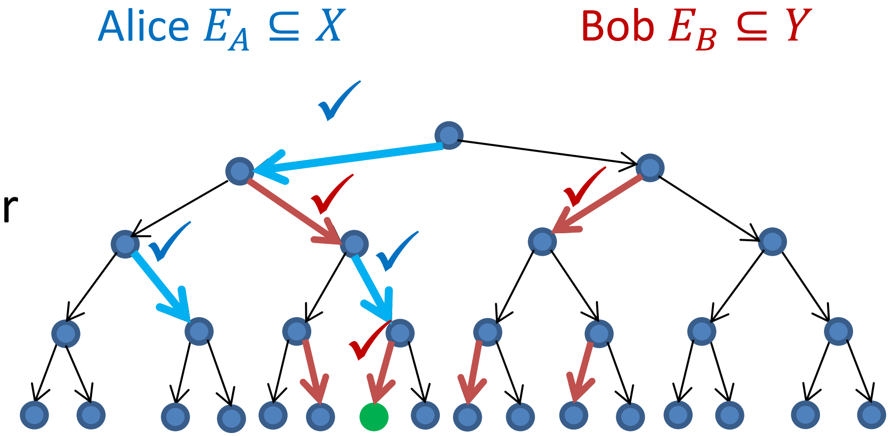
**Analysis:**

Two consecutive **uncorrupted** rounds

(1) the common path in $E_A \cup E_B$ grows, or

(2) both Alice and Bob add one vote to

the correct leaf




At most N/2 (1/2-2$\epsilon$) bad pairs ➔at least N/2 (1/2+2$\epsilon$) good pairs

At most n≤N$\epsilon$ good pairs for growing ➔ at least N/2 (1/2+$\epsilon$) good votes.

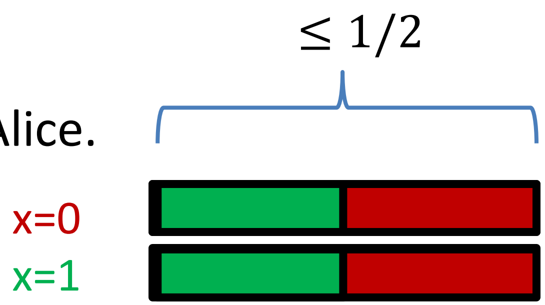
# Why 1/4 seems best possible?

Exchange problem:

Alice gets $x \in \{0,1\}$, Bob gets $y \in \{0,1\}$. Learn the other one's input.

Adversary:

- Take the party that sends less than ½ of the time, say Alice.
- Change ½ of Alice's transmissions.
- Bob cannot distinguish whether Alice has 0 or 1.




<u>Catch</u>**:** Assumes the party who sends less than ½ is fixed (independent of errors)

True if non-adaptive.

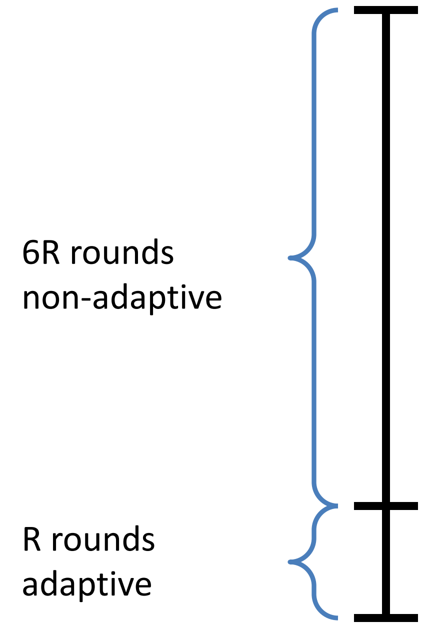
**Non-adaptive**: it's fixed a priori who sends in each round.

# Adaptivity

Adaptivity let's us improve the tolerable error-rate to $2/7 - \epsilon$.

**Exchange prob.:** Alice gets $x \in \{0,1\}$, Bob gets $y \in \{0,1\}$.

Learn the other one's input.




Use $N = 7R$ rounds, $R = O(1/\epsilon)$.

**Part 1: 6R rounds, non-adaptive**

Alice sends in odd rounds, Bob in even rounds, each 3R times.

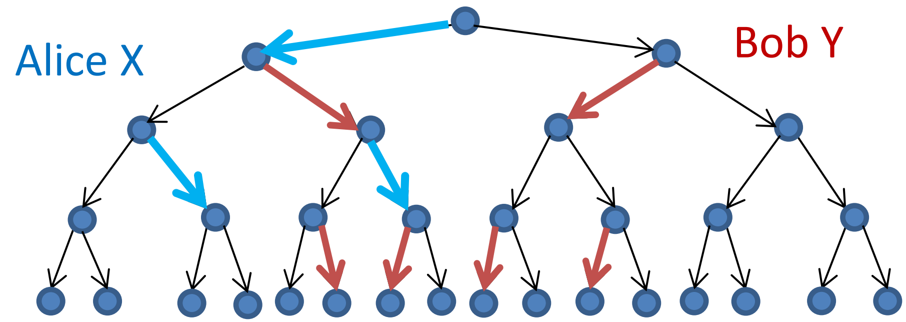
**Part 2: R rounds, one adaptive decision**

If among the 3R receptions in the first part, at least 2R rounds say 0 (or at least 2R rounds say 1), it is correct ("safe"); then just send. Otherwise, just listen.

At least one party will decode safely in the first part

Only one party will listen in the last R rounds.

# Tolerating error-rate $2/7 - \epsilon$ Adaptively




Take $N=7R$ rounds, for $R=O(n/\epsilon)$

Alice keeps $E_A \subseteq X$ , Bob keeps $E_B \subseteq Y$

**Alice's Algorithm:**

**Part 1: 6R rounds, non-adaptive** **--** send in odd rounds, listen in even rounds

<u>Sending round</u>: send a one-symbol indicating $E_A$

<u>Receiving round</u>: suppose received $E'_B$; ignore if it looks "invalid".

If $E_A \cup E'_B$ ends at a leaf v, add one vote to v.

Otherwise, if $E_A \cup E'_B$ can be extended along X via an edge e, let $E_A = E_A \cup \{e\}$.

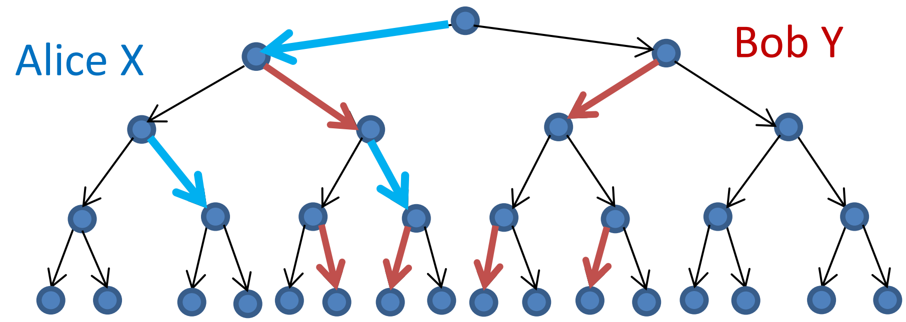
**Part 2: R rounds, one adaptive decision**

If there is a leaf that has *all except R votes*, "safe" to decode ➔ always send $E_A$

Otherwise, always listen. Each round add a vote to the leaf at the end of $E_A \cup E'_B$

# Tolerating error-rate 2/7 - $\epsilon$ Adaptively

N=7R rounds, for R=O(n/$\epsilon$)




**Part 2: R rounds, one adaptive decision:**

If there is a leaf that has all except R votes, "safe" ➔ always send $E_A$

Otherwise, always listen. Each round add a vote to the leaf at the end of $E_A \cup E'_B$

**Analysis:**

- "Safe" is indeed safe.
- At least one party is safe ➔ at most one listens.
- The listening party will also decode correctly.

# Tolerating error-rate $2/7 - \epsilon$ Adaptively

So far, N=O(n) rounds with alph. size O(n) bits

**Moving to O(1) alphabet size**

- Send over edge sets $E_A$ and $E_B$ with ($1- \epsilon$)-distance ECC using O(n) symbols
- List decode on the receiver side, add all results to the edge set
- For voting, do a soft decoding

A code for error-rate $2/7- \epsilon$, comm. comp. $N=O(n^2)$ rounds with alph. size O(1), and comput. comp. $\tilde{O}(n^2)$.

# Model Subtlety with Adaptivity

What's received when parties both listen or send in one round?

✓ A sending party does not receive anything.

Both listening is subtle: If both receive silence, they have an uncorrupted communication medium.

In the non-adaptive setting, avoided by design: no alg. should let both listen.

In adaptive, it happens unavoidably.

Fix: let the adversary decide what's received when both parties listen.

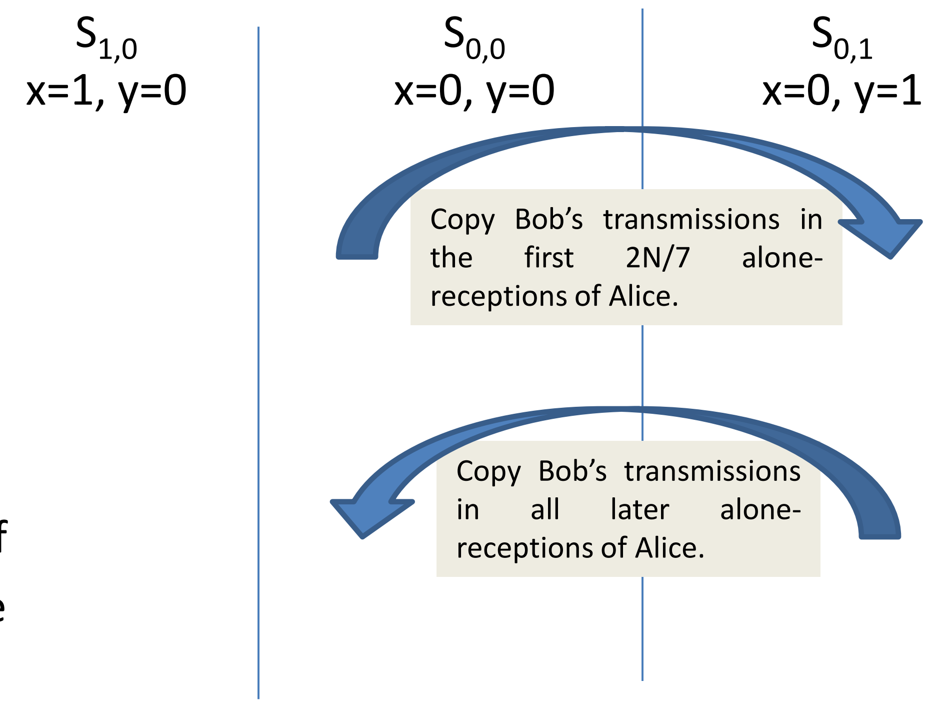
Prevents info. exchange in such rounds

# Optimality of 2/7

Take any protocol, say it uses N rounds.

<u>*Special* scenario</u>: whenever have 0, first 2N/7 alone-receptions will look as if the other party has 0, the later alone-receptions look as if the other party has 1.

Let $x_A$ and $x_B$ respectively be the number of receptions of Alice and Bob when they are (each) in the *special* scenario.



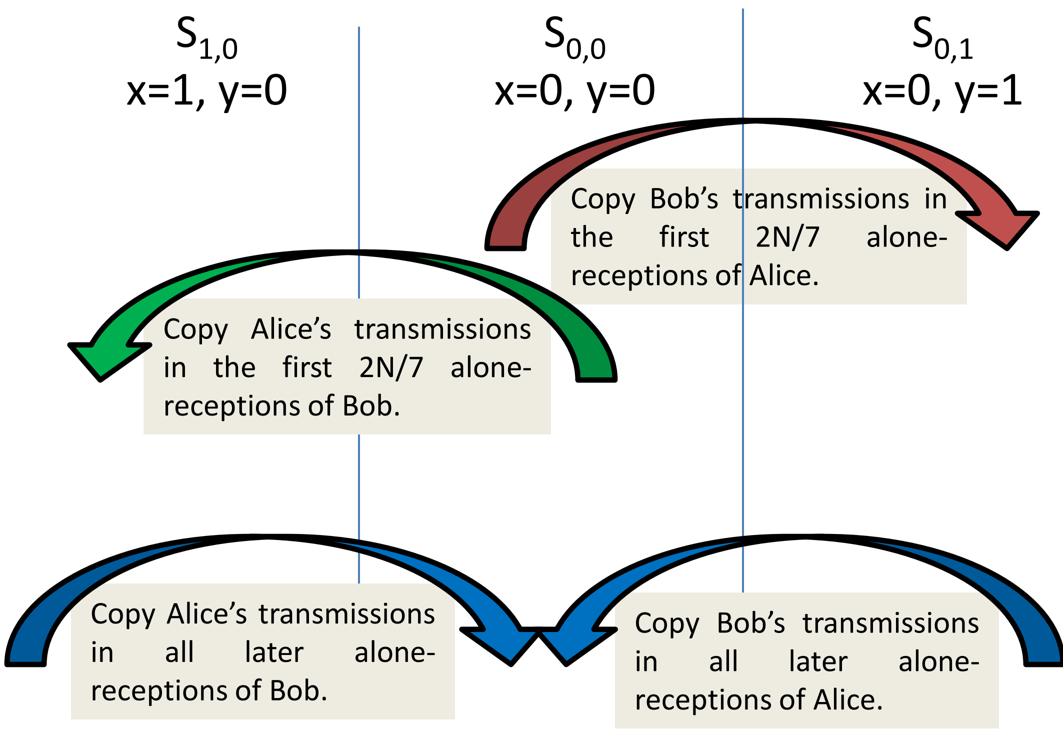

If $x_A \leq \frac{4N}{7}$, trick Alice. First 2N/7 alone-receptions, copy Bob's transmissions from $S_{0,0}$ to $S_{0,1}$. Remaining alone-receptions, copy Bob's transmission from $S_{0,1}$ to $S_{0,0}$.

If $x_B \leq \frac{4N}{7}$, do the same trick on Bob.

# Optimality of 2/7

*Special* scenario: whenever have 0, first 2N/7 alone-receptions will look as if the other party has 0, the later alone-receptions look as if the other party has 1.

Let $x_A$ and $x_B$ respectively be the number of receptions of Alice and Bob when they are (each) in the *special* scenario.




If $x_A > \frac{4N}{7}$ and $x_B > \frac{4N}{7}$ ➔ at least N/7 overlap ➔ each have less than 3N/7 alone reception, trick both, Alice between $S_{0,0}$ and $S_{0,1}$ and Bob between $S_{0,0}$ to $S_{1,0}$

# Conclusion & Open Problems

✓2/7 is the optimal (sharp) threshold on the tolerable error-rate .

✓2/3 is the optimal threshold if parties have (hidden) shared randomness,

✓1/2 is the optimal threshold if parties want to just list decode.

**Newer results:**

Optimal tolerable error-rates, N=O(n) comm. rounds, and comput. comp. Õ(n).
Randomized with fail. prob. $2^{-\Theta(n)}$.

**Open questions:**

(1) Explicit deterministic construction? The above randomized code also gives a non-uniform deterministic version.

(2) Optimal communication complexity/rate for each error-rate?

Thank you