An Improved Lower Bound on Rate for
Variable-length Codes with Active Feedback
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Main result: Improved lower bound on maximum rate of variable-length feedback codes at short blocklengths
Previous lower bound [Polyanskiy, Poor and Verdú, 2011]: stop-feedback codes, left large gap to upper bound
New approach: “active” feedback to confirm receiver’s estimate
Numerical results provided for BSC

Stop-feedback

Stop-feedback VLF code:
Tx ignores feedback except to learn when Rx stops transmission (decodes)
Encoder outputs $X_e = f_e(U, W)$
Also called decision feedback (ACK/NACK from Rx)

Finite-blocklength regime:
Feedback improves the maximum rate at short blocklengths compared to no-feedback case. (Fig. 1)
Large gap between lower (achievability) and upper (converse) bounds on rate.
Best achievable result for DMCs based on stop-feedback codes – Doesn’t consider what receiver knows!

"Active" Feedback

Transmitter uses feedback to refine receiver’s tentative estimate.
In general, $f_e(U, W, y^{n-1}) \neq f_e(U, W, \hat{y}^{n-1})$, when $y^{n-1} \neq \hat{y}^{n-1}$
Channel coding is a specific case of active sequential hypothesis testing [Naghshvar and Javidi, 2012].
Benefit of active feedback called adaptivity gain.
Active feedback also called information feedback.

Improved Lower Bound

Proposed scheme:
Decoder feeds back estimate $\hat{x}^n$ once $i(x^n; \hat{x}^n) \geq \gamma$ for some $x^n$
Tx uses $N$ forward symbols to confirm (ACK) or deny (NACK) estimate
Start over if Rx sends NACK, stop when Rx sends ACK
$P[n \rightarrow a] = P[\text{NACK decoded as ACK}]$
$P[a \rightarrow n] = P(\text{ACK decoded as NACK})$
$P(\text{NACK}) = P(\text{Rx decodes NACK})$
   $= P[n \rightarrow n]P(\text{Rx est. wrong}) + P[a \rightarrow n]P(\text{Rx est. correct})$
   $\leq P[n \rightarrow n](M-1)P[\tau \leq \tau_0] + P[a \rightarrow n]$

Theorem: Improved Achievability for Active Feedback
For a scalar $\gamma > 0$ and integer $N > 0$, there exists an $(I, M, e)$ VLF code satisfying

\[
I \leq \frac{E[x]}{1 - \frac{P[NACK]}{P[NACK] + (M-1)P[NACK]P[tau] + P[a \rightarrow n]}}
\]

Proof: Similar to stop-feedback proof.
Numerical evaluation (Fig. 2) requires optimization over $\gamma$, $N$, and threshold $\tau_0$ (threshold for skewed hypothesis test of confirmation block at Rx), for fixed $M$ and $e$.

Discussion
Can do better by refining Rx estimate sequentially, not just at $\tau$
Starting over after NACK is costly in terms of latency
Still need to find “good” codes
There may be encoder complexity challenges

References:

Stop-feedback Bound

Theorem: (Stop-feedback) Achievability [PPV’11, Thm. 3]
For a scalar $\gamma > 0$, there exists an $(I, M, e)$ VLF code satisfying
\[
I \leq \frac{E[x]}{1 - \frac{P[NACK]}{P[NACK] + (M-1)P[NACK]P[tau] + P[a \rightarrow n]}}
\]

Proof: Random coding argument.

VLF Codes

An $(I, M, e)$ variable-length feedback (VLF) code consists of [Polyanskiy, Poor and Verdú, 2011]:
Message $W \in \{1, 2, ..., M\}$
Average blocklength $l = E[I] = I$
$\tau$ is a stopping time of the filtration $\sigma(U, Y_1, Y_2, ...)$
$U$ is common randomness revealed to both Tx and Rx
Encoder outputs $X_e = f_e(U, W, Y_1, Y_2, ...)$
Memoryless channel $P(Y | X_1, X_2, ... X_e) = P(Y | X)$
Decoder’s estimates $\hat{g}_a(U, Y_1, Y_2)$
Decoder’s final decision $W = g_a(U, Y_1, Y_2)$
Average probability of error $e$ s.t. $P[\hat{W} \neq W] \leq e$
Code rate is $[\log M] / l$