The Entropic Framework

As in any inference problem we start by defining our subject matter, assumptions, prior beliefs, and constraints. Together these will form our framework of inferences, the Entropic Framework (EF).

Subject Matter:

1. The particle exists and has a definite position, $x$, but in general this position is known.
2. The particle lives in some space $X$, endowed with a metric, $d$.
3. Change happens. A particle at position $x$, will "jump" to some new position $x'$.
4. There exist other "things" (extra variables) which influence the particle. These variables take a value $y$ and live in some space $Y$, with measure $q$.
5. The value of those variables take of consequence the dynamics of the particle. It is one's own uncertainty, $p(x,y)$, that they contribute to the dynamic, more specifically through their $p(x|y)$ which forms the subject matter (see Figure 1).

By adding an extra variable $Y$ to our space $X$, our new space $X \times Y$ is formed.

Since both the position variable and the extra variables, are relevant to the ontology, the joint variables, by assigning a $p(y|x)$ to each $x$, we have

$$P(x,y) = \frac{p(x,y)}{\int p(x,y) dy}$$

This is in a measure on both $X$ and the extra, space $Y$. The relevant $y$, $p(x,y)$, is relevant to the ontology, the joint space $X \times Y$ is formed.

The Entropic Framework outlined here is robust yet malleable enough to generate several variations on the EF.

The Generating Dynamics by Varying an Entropic Framework

In the Entropic Framework, we shall take the observable properties of any system and let the system evolve from one state to another. The dynamics of the system is described by an energy function, $H(x,y)$, which is the Hamiltonian of the system. The dynamics of the system is then given by the gradient of $H$ with respect to the state variables $x$ and $y$.

$$\frac{d}{dt}x = -\frac{\partial H}{\partial y}, \quad \frac{d}{dt}y = \frac{\partial H}{\partial x}$$

The EF Variation 1: Diffusion

In this variation, the energy function $H(x,y)$ is chosen to be a quadratic function of the form

$$H(x,y) = \frac{1}{2} m \dot{x}^2 + V(x)$$

where $m$ is the mass of the particle, $\dot{x}$ is its velocity, and $V(x)$ is the potential energy at position $x$. The dynamics of the system are then given by

$$\frac{d}{dt}x = -\frac{\partial}{\partial y} \left( \frac{1}{2} m \dot{x}^2 + V(x) \right) = -\frac{\partial}{\partial y} V(x)$$

When $V(x)$ is a potential energy function, the dynamics of the system are then given by

$$\frac{d}{dt}x = -\frac{\partial}{\partial y} V(x)$$

The EF Variation 2: Quantum Mechanics

In this variation, the energy function $H(x,y)$ is chosen to be a complex function of the form

$$H(x,y) = \frac{1}{2} m \dot{x}^2 + V(x) + i S$$

where $S$ is the entropy of the system. The dynamics of the system are then given by

$$\frac{d}{dt}x = -\frac{\partial}{\partial y} \left( \frac{1}{2} m \dot{x}^2 + V(x) + i S \right) = -\frac{\partial}{\partial y} V(x) - i \frac{\partial}{\partial y} S$$

When $S$ is a constant, the dynamics of the system are then given by

$$\frac{d}{dt}x = -\frac{\partial}{\partial y} V(x) - i \lambda$$

where $\lambda$ is a constant.

In this variation, the dynamics of the system are then given by

$$\frac{d}{dt}x = -\frac{\partial}{\partial y} V(x) - i \lambda$$

which is a complex equation describing the dynamics of the system. The entropy $S$ is a measure of the uncertainty in the system, and it is the imaginary part of $H$ that gives rise to the complex dynamics.

The EF Variation 3: Hybrid Mechanics

In this variation, the energy function $H(x,y)$ is chosen to be a hybrid function of the form

$$H(x,y) = \frac{1}{2} m \dot{x}^2 + V(x) + i S + \sigma$$

where $\sigma$ is a constant. The dynamics of the system are then given by

$$\frac{d}{dt}x = -\frac{\partial}{\partial y} \left( \frac{1}{2} m \dot{x}^2 + V(x) + i S + \sigma \right) = -\frac{\partial}{\partial y} V(x) - i \frac{\partial}{\partial y} S - \sigma$$

When $S$ is a constant, the dynamics of the system are then given by

$$\frac{d}{dt}x = -\frac{\partial}{\partial y} V(x) - i \lambda - \sigma$$

which is a complex equation describing the dynamics of the system. The hybrid term $\sigma$ is a measure of the uncertainty in the system, and it is the imaginary part of $H$ that gives rise to the complex dynamics.

Final Remarks

The Entropic Framework outlined here is robust yet malleable enough to generate several variations on the EF. The dynamics of the system are then given by

$$\frac{d}{dt}x = -\frac{\partial}{\partial y} \left( \frac{1}{2} m \dot{x}^2 + V(x) + i S + \sigma \right) = -\frac{\partial}{\partial y} V(x) - i \frac{\partial}{\partial y} S - \sigma$$

which is a complex equation describing the dynamics of the system. The hybrid term $\sigma$ is a measure of the uncertainty in the system, and it is the imaginary part of $H$ that gives rise to the complex dynamics.

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Select References