Geometric WOM codes and adaptations to multilevel flash codes

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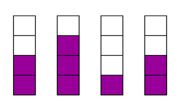
What is flash memory?

- A non-volatile storage medium used in many technologies: USB drives, digital cameras, phones, and hybrid computer hard drives.
- The memory is organized into blocks of $\sim 10^5$ cells, each of which can be charged up to one of q levels.



- Increasing cell charge is easy, decreasing is costly.
- Codes for flash memories generalize Write Once Memory (WOM) codes (case when $q = 2$).

WOM goals and notation

- maximize the number of rewrites before erasing
- incorporate error correction
- construct low-complexity, high-rate codes

Notation:

- $\mathcal{C} = \langle v_1, \ldots, v_t \rangle / n$ is a binary t-write WOM code on n cells, representing v_i messages on the i^{th} write.
- $\langle v \rangle^t / n$ denotes a code where $v_1 = v_2 = \cdots = v_t$.
- The *rate* of $\mathcal{C}$ is $\dfrac{\log_2(v_1 \ldots v_t)}{n}$.

Example: Rivest-Shamir WOM code

This code maps 2 information bits to 3 coded bits and tolerates two writes.

Info.	1^{st} write	2^{nd} write
00	**000**	**111**
01	**100**	**011**
10	**010**	**101**
11	**001**	**110**

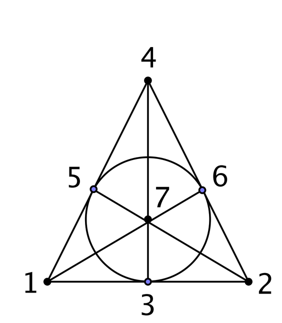
The sequence $\boxed{11} \longrightarrow \boxed{10}$ would be written in the memory as

$$\boxed{001} \longrightarrow \boxed{101}$$

$PG(m, 2)$ and the Hamming code

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Minimum weight words in the $[7, 4]$ Hamming code.

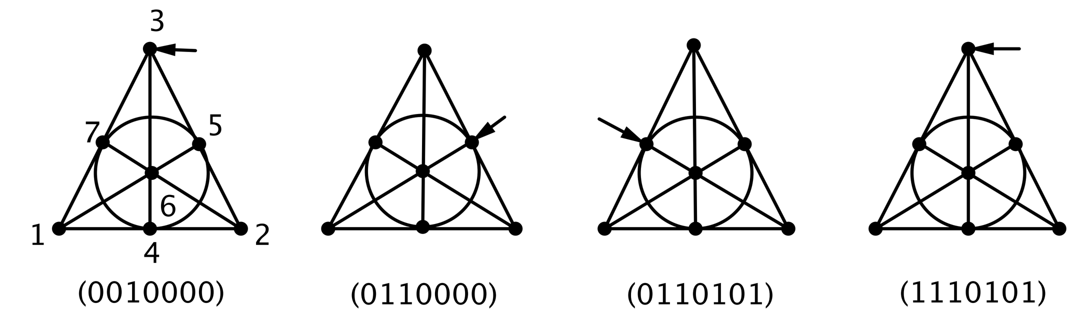


Lines in $PG(2, 2)$

Merkx's $\langle 7 \rangle^4 / 7$ WOM code using $PG(m, 2)$

- Merkx [?] constructed WOM codes based on finite projective geometries over $\mathbb{F}_2$.

$$\boxed{\text{Messages}} \leftrightarrows \boxed{\text{points in PG(m,2)}}$$



Four writes of the Merkx $PG(2, 2)$ WOM code.

- WOM codewords are one error from a binary Hamming codeword; the location of the error indicates the point that corresponds to the information message.

Reed-Muller codes

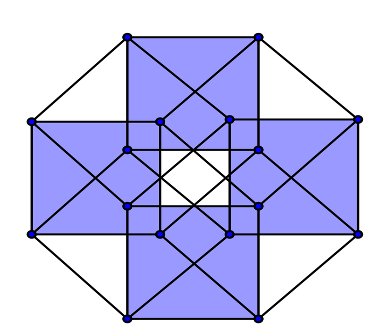
- Let $F = \mathbb{F}_2$, V be a vector space of dimension m over F, and F^V the set of functions from $V \longrightarrow F$.

The *Reed-Muller code of order* r *and length* 2^m, $\mathcal{R}(r, m)$ is the subspace of F^V that consists of all polynomial functions of degree $\leq r$:

$$\mathcal{R}(r, m) = \left\langle \prod_{i \in I} x_i | I \subseteq \{1, 2, \ldots, m\}, 0 \leq |I| \leq r \right\rangle.$$

Another geometric connection

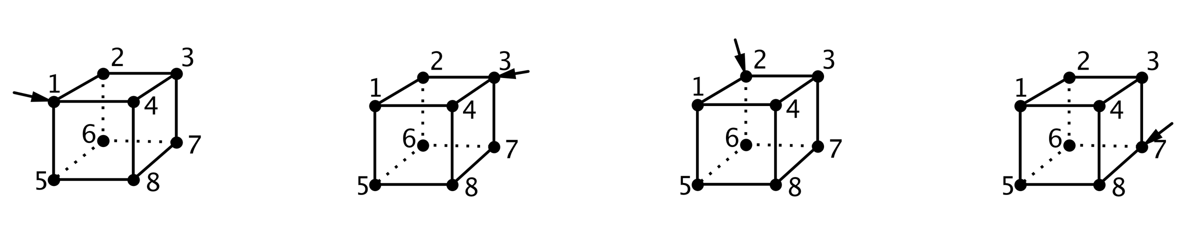
Minimum-weight generators of the $\mathcal{R}(2, 4)$ code correspond to 2-flats in $EG(4, 2)$.



In general $\mathcal{R}(m - 2, m) \rightleftarrows EG(m, 2)$.

New WOM codes

The following is an example of four writes in the $\langle 8, 8, 8, 4 \rangle / 8$ WOM code from $EG(3, 2)$:



- The WOM code corresponding to $EG(4, 2)$ has parameters

$$\langle 16, 16, 16, 12, 8, 8, 8, 4 \rangle / \ 16.$$

Result

Proposition: $EG(m, 2)$ gives rise to a WOM code with $4(m - 2)$ writes and parameters

$$\langle \underbrace{2^m, 2^m, 2^m, 2^m - 4}, \underbrace{2^{m-1}, 2^{m-1}, 2^{m-1}, 2^{m-1} - 4}, \ldots, \underbrace{8, 8, 8, 4} \rangle / \ 2^m$$

Proof idea:

- Find a hyperplane that contains the first four information points, and use the $EG(3, 2)$ code on a 3-flat.
- Set all other points in the hyperplane to one, and use the $EG(m - 1, 2)$ code on the remaining points.

Rate comparison and remarks

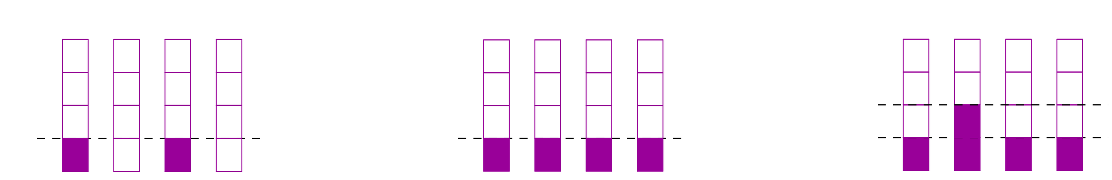
Code	length	rate
$PG(2, 2)$	**7**	**1.60**
$EG(3, 2)$	**8**	**1.38**
$PG(3, 2)$	**15**	**1.82**
$EG(4, 2)$	**16**	**1.66**
$PG(4, 2)$	**31**	**1.60**
$EG(5, 2)$	**32**	**1.50**

Merkx PG codes have higher rates; EG codes have simple encoding and decoding as well as code lengths that are powers of 2.

Using binary WOM codes on multilevel cells

Flash memory cells on $q > 2$-levels motivates coding strategies for 'generalized' WOMs. Reapplication of binary WOM codes provides a basis for comparison.

The complement scheme:



Use a binary WOM code on the cells.

After binary writes are exhausted, bump all cell levels to 1.

Use binary WOM on levels between 1 and 2.

Strategies for adapting to q-level cells

R-S code on $q = 3$ levels using complement scheme:

Information	1^{st} write	2^{nd} write	3^{rd} write	4^{th} write
00	000	111	111	222
01	100	011	211	122
10	010	101	121	212
11	001	110	112	221

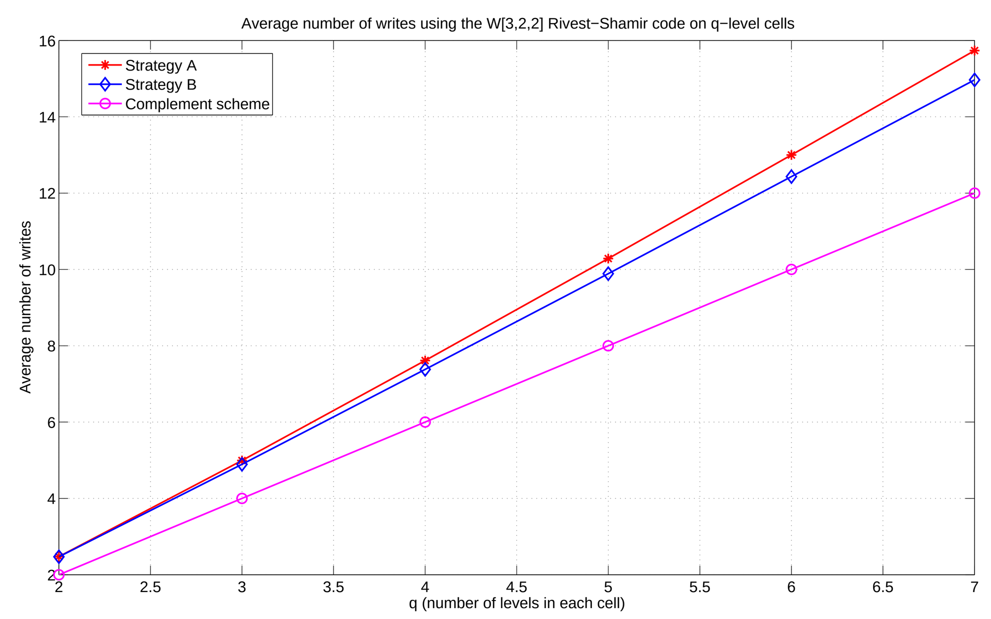
Improved schemes:

Use a WOM code $\mathcal{C}$ by finding a q-ary word c that is component-wise $\geq$ current memory contents, with $c(mod 2) \in \mathcal{C}$; decodes to message. Choose c that

- *Strategy A*: minimizes the number of cells that are increased.
- *Strategy B*: minimizes the highest level cell (distribute increases evenly among all cells).

Comparison of average write performance

Theorem: Let $\mathcal{C}$ be an $\langle v \rangle^t / n$ binary WOM code. Then, the guaranteed number of writes by applying either strategy A or strategy B to $\mathcal{C}$ on q-level flash cells is at least $(q - 1)t$.



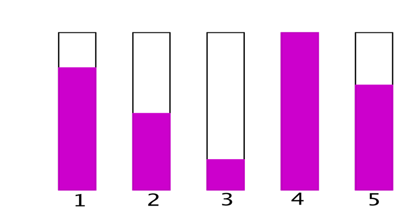
The average write performance of Strategies A, B, and the complement scheme for the Rivest-Shamir code, simulated on 10^5 random message sequences to record the number of writes.

Conclusions

- Showed how EGs can be used to obtain a new family of WOM codes with structure useful in schemes that require component WOM codes.
- Introduced strategies for adapting WOM codes to multilevel cells.

Future work:

- Determine qualities of the underlying WOM code that cause either Strategy to perform better.
- Quantify average performance as $q \longrightarrow \infty$.
- Construct binary or multilevel flash codes using q-ary codes or other discrete structures.
- Coding for the rank modulation scheme.



The permutation induced by the cell levels on the left is $(4, 1, 5, 2, 3)$. In general, the permutation is $(x_1, x_2, \ldots, x_n)$, where x_1 is the number corresponding to the cell with the highest level, x_2 the cell with the second highest level, etc.

Select references:

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