Fading Gaussian broadcast channels with receiver only channel state information

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Problem Setup

- Two receiver broadcast channel

$$Y_1 = h_1 X + Z_1$$

$$Y_2 = h_2 X + Z_2$$

- Fading - random variables h_1 and h_2 known only at receivers.

  Unit variance Gaussian additive noise Z_1, Z_2.

  Transmission power constraint P.

  The sender X wishes to send a common message M_1 to receiver Y_1 and M_2 to receiver Y_2.

- When h1 and h2 are fixed, channel is degraded; Superposition coding is optimal.
- Superposition coding inner bound

  A rate pair (R_1, R_2) is achievable for the DM-BC $p(y_1, y_2|x)$ if

$$R_1 < I(X; Y_1|U)$$

$$R_2 < I(U; Y_2)$$

$$R_1 + R_2 < I(X; Y_1)$$

  for some pmf $p(u,x)$.

- Superposition coding is optimal for degraded BCs, less noisy BCs, and more capable BCs.
- If $h_1 > h_2$, $R_1 < C(a|h_1|^2 P)$, $R_2 < C(\frac{|h2|^2(1-a)P}{|h2|^2 aP + 1})$ is the capacity region.

- If h_1 and h_2 are random variables but known at the transmitter, the broadcast channel is parallel with degraded components; capacity is known.
- The question is...

  What is the capacity of the fading BC when fading states are known only at the receivers?

  The difficulty is that the encoder does not know in which direction the channel is degraded during each coherence time interval.

Achievable schemes

- Superposition coding [2,3]

  $X = X_1 + X_2$. $X_1 \sim N(0, aP)$, $X_2 \sim N(0, (1-a)P)$

  If $C_1(P) - C_2(P) \geqq C_1(\alpha P) - C_2(\alpha P)$ for all $\alpha \in [0,1]$, where $C_i(P) = E[\log(1 + |h_i|^2 P/N)]$, successive cancellation decoding can be performed.

$$R_2 < E[log(1 + \frac{|h_2|^2(1-a)P)}{N + |h_2|^2 aP})], R_1 < E[log(1 + \frac{|h_1|^2 aP}{N})]$$

  is achievable.

Simplify the problem.

- Assume there are only two fading states. $h_{11} > h_{12}$, $h_{21} < h_{22}$.

	h_1	h_2
State 1 (p_1)	h_{11}	h_{12}
State 2 (p_2)	h_{21}	h_{22}

Achievable schemes

- Marton's Inner Bound for Fading DM-BC

  (R_1, R_2) is achievable for the DM-BC $p(y_1, y_2|x)$ if

$$R_1 < I(U_1; Y_1|S)$$

$$R_2 < I(U_2; Y_2|S)$$

$$R_1 + R_2 < I(U_1; Y_1|S) + I(U_2; Y_2|S) - I(U_1; U_2)$$

  for some $p(u_1, u_2)$, Rx CSI $S = (h_1, h_2)$, and function $x(u_1, u_2)$.

- $X = X_1 + X_2$, $X_1 \sim N(0, aP)$, $X_2 \sim N(0, (1-a)P)$, $U_1 = X_1$, $U_2 = U_1 + \beta X_2$.
- Achievable rates

$$R_1 = E[log(1 + \frac{|h_1|^2 aP}{|h_1|^2(1-a)P + 1})], R_2 = E[log(\frac{(1-a)P(|h_2|^2 P + 1)}{a(1-a)|h_2|^2 P^2 (1-B)^2 + ((1-a)P + B^2 aP)})]$$

$$R_2' = E[log(1 + \frac{|h_2|^2 aP}{|h_2|^2(1-a)P + 1})], R_1' = E[log(\frac{(1-a)P(|h_1|^2 P + 1)}{a(1-a)|h_1|^2 P^2 (1-B)^2 + ((1-a)P + B^2 aP)})]$$

  (R1,R2) ∪(R1',R2') for a,B ∈[0,1] are achievable by Marton's coding.

Plan

- Simulate Marton's coding and superposition coding. See if Marton's coding is better than superposition coding when both schemes can be used.
- Find the conditions for superposition coding.

Related works

- Jafarian, A.; Vishwanath, S. 2008[4]

  They investigate inner/upper bounds on the capacity of one-sided to user Gaussian fading broadcast channels and show that the inner and upper bounds meet under special cases.

- Tse, D.; Yates, R.; Zang Li, 2008[5]

  They propose a layered erasure broadcast channel to approximate the Gaussian fading channel and determined its capacity region exactly.

  Using the insights from the erasure model, they derive a new outer bound to the Gaussian fading BC capacity region and demonstrated a scheme that achieves rates within 6.386 bits/s/Hz per user to the outer bound.

References

[1] A. El Gamal and Young-Han Kim, "Network Information Theory," Cambridge University Press, 2011

[2] D. Tuninetti and S. Shamai, "On two user fading Gaussian broadcast channels with perfect channel state information at the receivers," in Proceedings of IEEE Int. Symp. Inform. Theory, 2003.

[3] D. Tuninetti and S. Shamai, "Fading Gaussian broadcast channels with state information at the receivers," DIMACS Series in Discrete Mathematics and Theoretical Computer Science, 2003.

[4] Jafarian, A.; Vishwanath, S.; , "On the capacity of one-sided two user Gaussian fading broadcast channels," Information Theory and Applications Workshop, 2008 , vol., no., pp.428-432, Jan. 27 2008-Feb. 1 2008

[5] Tse, D.; Yates, R.; Zang Li; , "Fading broadcast channels with state information at the receivers," Communication, Control, and Computing, 2008 46th Annual Allerton Conference on , vol., no., pp.221-227, 23-26 Sept. 2008